Goldstein
7.6

The amplitudes as measured by 2 observers are

$$f(\vec{x},t) \propto e^{i(\vec{k}\cdot\vec{x} - wt)}, \quad f(\vec{x}',t') \propto e^{i(\vec{k}'\cdot\vec{x}' - w't')}$$

we demand the ampludes be equal

$$e^{i(\vec{k}\cdot\vec{x} - wt)} = e^{i(\vec{k}'\cdot\vec{x}' - w't')}$$

$$\Rightarrow \quad \vec{k}\cdot\vec{x} - wt = \vec{k}'\cdot\vec{x}' - w't'$$

Introduce 4-vectors $x^\mu = (ct, \vec{x})$, $k^\mu = (\frac{w}{c}, \vec{k})$

This suggests $x^\mu k^\nu g_{\mu\nu} = x^\mu k_\mu$ is Lorentz invariant,

$\boxed{\text{consequently, } k^\mu \text{ is covariant.}}$

By covariance of k^μ, we know it follows Lorentz transformati
Recall the transformation on spacetime coordinates:

$$\begin{cases} ct' = \gamma(ct - \vec{\beta}\cdot\vec{r}) \\ \vec{r}' = \vec{r} + \dfrac{(\vec{\beta}\cdot\vec{r})\vec{\beta}(\gamma-1)}{\beta^2} - \vec{\beta}\gamma ct \end{cases}$$

This suggests the transformation for k^μ:

$$k^{0\prime} = \gamma(k^0 - \vec{\beta}\cdot\vec{k})$$

$$\Rightarrow \quad \frac{w'}{c} = \gamma\left(\frac{w}{c} - \vec{\beta}\cdot\vec{k}\right),$$

$$\boxed{w' = \gamma(w - \vec{v}\cdot\vec{k})}$$

This is not the familiar form of relativistic doppler; to get such, do assume

$$\vec{\beta} \cdot \vec{k} = \beta k, \quad \text{(boosts allign with } \vec{k}\text{)}$$

Then $\quad \omega' = \gamma(\omega - c\beta k)$

use dispersion $\quad \frac{\omega}{k} = c \quad , \quad k = \frac{\omega}{c}$

$$\omega' = \gamma(\omega - \beta\omega)$$

$$= \omega \frac{(1-\beta)}{\sqrt{1-\beta^2}}$$

$$= \omega \sqrt{\frac{\cancel{(1-\beta)}(1-\beta)}{\cancel{(1-\beta)}(1+\beta)}}$$

$$\boxed{\omega' = \omega \sqrt{\frac{(1-\beta)}{(1+\beta)}}}$$

Davidson Cheng

2. 8. 2024